

## A hysteresis-based damage parameter for notched composite laminates subjected to cyclic loading

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### Introduction

With the increasing use of polymer matrix composites in aircraft primary structure, it is critical that damage propagation during operation is accurately detected and monitored. Unlike metals, composite materials can sustain damage that is not readily detectable by visual or nondestructive inspection. Fiber optic strain sensors have performed satisfactorily in laboratory specimens but require the installation of a large number of sensors in real structures to monitor localized damage zones. In addition to complexity and installation issues, strain sensors need to be located near potential areas of damage that are not easy to determine a priori. This paper introduces a more sensitive damage parameter that can be readily measured in practice and takes into account the varied damage mechanisms present in composites [1]. These include matrix crazing, microcracking, fibers breakage, delamination, interfacial failure and fiber pullout.

A common measure of damage has been stiffness degradation in which the stiffness of the laminate is measured over each fatigue cycle and compared with the initial stiffness [2]. The moduli used for monitoring damage have been various: elastic (loading or unloading) [3–5], absolute, storage, loss, and total fatigue moduli [6]. The damage parameter is a function of the ratio of the modulus at the  $n$ th cycle to the initial modulus, and is written as,

$$D_n = 1 - \frac{E_n}{E_i}, \quad (1)$$

where  $D_n$  and  $E_n$  are the damage and modulus, respectively, at the  $n$ th cycle, and  $E_i$  is the initial modulus. Typically, the material is considered to have failed when the damage reaches a certain value, for example, 0.15–0.30. The secant moduli criterion is similar except the specimen is considered to have failed when the secant modulus at the  $n$ th cycle is within the scatter band of the static failure secant modulus [7].

Stress or strain degradation can also be used as the damage parameter in load or stroke-controlled fatigue [8]. An empirical stress-based fatigue life prediction, for example, uses the equation [9]:

$$\frac{\sigma_{\max}}{\sigma_t} = 1 - B \log(2N_f), \quad (2)$$

where  $\sigma_{\max}$  is the maximum cyclic stress,  $\sigma_t$  is the tensile fracture strength,  $N_f$  is the number of cycles to failure and  $B$  is a constant.

A strain energy-based degradation model has also been proposed in which stiffness and stress or strain degradation are combined to form a criterion for damage-tolerance [8, 10]. An example of this approach is to use the empirical equation [10]:

$$\frac{dU_e}{dN} = a \left( \frac{\varepsilon_{\max}}{\varepsilon_{\text{ult}}} \right)^b, \quad (3)$$

where  $dU_e/dN$  is the strain energy per cycle,  $\varepsilon_{\max}$  is the maximum cyclic stress,  $\varepsilon_{\text{ult}}$  is the ultimate strain and  $a$  and  $b$  are constants.

We propose instead that hysteresis per loading cycle, monitored continuously, is a more sensitive measure of damage in composites subjected to cyclic

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loading. Real time monitoring of hysteresis on a cycle-by-cycle basis is achievable today with high speed data logging and computational systems.

Hysteresis, which results from the phase lag between stress and strain, has been employed to provide a qualitative assessment of damage in composites. Various geometric features of the hysteresis loop have been used empirically as damage parameters. These include, for example, a “center of gravity” of the hysteresis loop, which evolves linearly with fatigue damage [11], or the area of the loop,  $W$ , that is a measure of energy dissipation [12–16].

We propose to use hysteresis dissipation energy as the basis for a damage parameter that is measured continuously during cyclic loading and is used to monitor and establish residual life. Our proposed damage parameter,  $D'$ , is given by

$$D'(n) = \frac{U' - U_i}{U_e}, \quad (4)$$

where  $D'$  is the damage at  $n$ th cycle,  $U'$  is the cyclic hysteresis energy,  $U_i$  is the initial hysteresis energy and  $U_e$  is the initial strain energy.

The numerator represents the hysteresis energy growth per cycle due to fatigue damage alone since the initial hysteresis energy,  $U_i$ , can be thought of as a background hysteresis associated with viscoelastic or other processes that create hysteresis, but are not associated with damage. The ratio of this damage-related hysteresis energy to the elastic strain energy,  $D'$ , is analogous to calculating a damping factor associated with fatigue damage and is more sensitive to detecting damage than stiffness degradation [17, 18]. In our approach,  $D'$  is obtained from real-time monitoring of hysteresis in a real structure during operation (health monitoring) without having to perform dynamic testing that require interruption of the loading, often not possible in service [18–20].

## Experimental

Our proposed damage parameter was evaluated experimentally by cyclic loading polymer matrix composite laminate [0/90] specimens and analyzing the load–displacement data. The specimens were constructed of 10 plies of E-glass prepreg laminated by compression molding 10 plies in a heated platen press and cured for 2 h at 177 °C (350 °F) in a heated platen press. The prepreg consists of 7781 Style 8-harness satin weave E-glass fabric with a fiber areal weight (FAW) of 300 g/m<sup>2</sup> pre-impregnated with YLA RS3 cyanate-ester resin

(YLA, Inc., Martinez, California, USA) with and a cured lamina or ply thickness of 200 μm (0.008”).

The cured laminate thickness ranged from 1.90 mm (0.075”) to 1.98 mm (0.078”). Fiber volume fraction of the laminates were calculated from thickness measurements and ranged from 0.62 to 0.64. The laminates were then cut into specimens, nominally 25 mm (1”) wide and 200 mm (8”) long, using a water cooled abrasive saw. A central 6.4 mm (0.250 inch) diameter hole was drilled using diamond abrasive drills to simulate holes commonly drilled in structural frames for inserts, bolting or conduits for wiring and piping.

The fatigue testing system consisted of a MTS (Eden Prairie, MN USA) 100 kN servohydraulic single actuator system that was retrofitted with a variable flow hydraulic supply and an Instron (Norwood, MA USA) Labtronic 8400 controller to improve reliability. The Instron controller was connected to a Windows XP computer via the GPIB bus and a custom program was coded in NI Labview, Version 7 to command the controller and perform data acquisition. Testing was done under load control in zero-tension with an R-ratio of 0.15, a sinusoidal waveform and a frequency of 3 Hz. One specimen was tested with a maximum stress of 168, 152,136 and 125 MPa. Two specimens were tested at a maximum stress of 115 MPa. The fatigue testing was done at room temperature and the low test frequency allows us to ignore specimen heating.

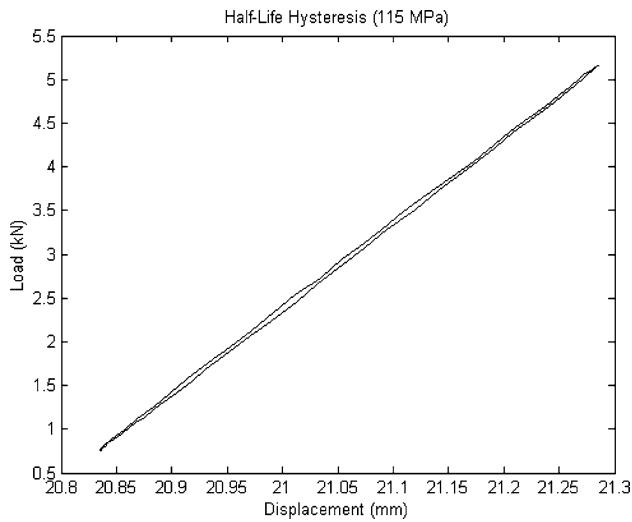
Load–displacement data were collected in real time at each cycle but data sampling rates have to be balanced between desired accuracy in calculating hysteresis and data storage limitations. A 5 ms data sampling rate allows for uninterrupted data capture for around 600,000 cycles and reasonable accuracy in calculating hysteresis area.

## Results

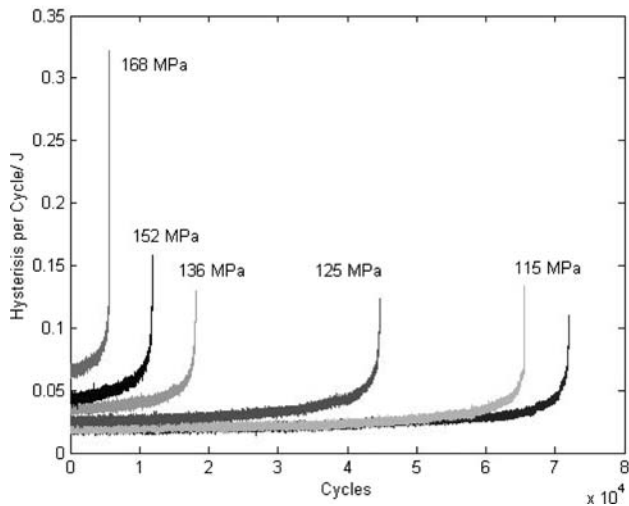
The experimental load and displacement data collected was analyzed using custom coded MATLAB. Hysteresis for each cycle is obtained using a simple trapezoidal summation of area between the cyclic loading and unloading paths. A typical hysteresis loop is shown in Fig. 1. The hysteresis per cycle for various peak cyclic stress levels is shown in Fig. 2.

## Discussion

When the damage parameter,  $D'$ , determined from Eq. 4 for each stress level, is plotted against the life



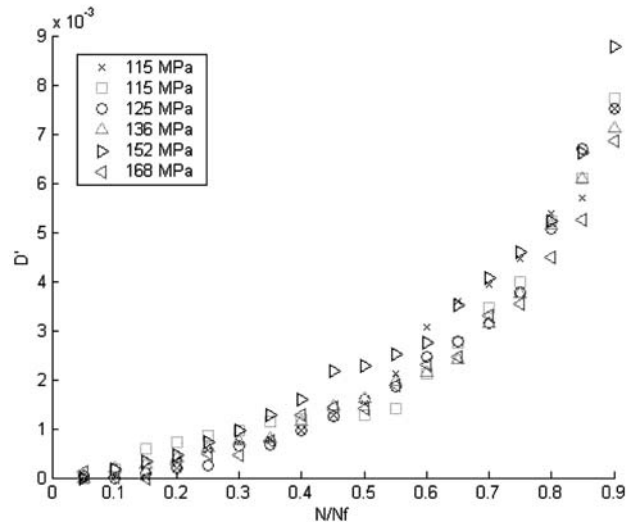
**Fig. 1** Sample hysteresis loop



**Fig. 2** Measured hysteresis per cycle at various stress levels

ratio,  $N/N_f$ , one obtains a master curve shown in Fig. 3. While the data of the two specimens at 115 MPa coincide fairly well, additional testing is in progress to establish the repeatability of monitoring the cyclic hysteresis and calculating  $D'$ . The initial hysteresis energy used in calculating  $D'$  is averaged over the first 20 cycles. The master curve is approximately linear between life ratios of 0.1–0.7 and can be used to predict the residual fatigue life by monitoring the rate of hysteresis growth.

Observations made during fatigue testing revealed visible damage starting to appear at a life ratio from 0.5 to 0.7 in the form of whitening at the edge of the hole, at the equator, and is due to fiber breakage. The observed damage continues to grow until the specimen fails catastrophically. Figure 4 shows such damage in a

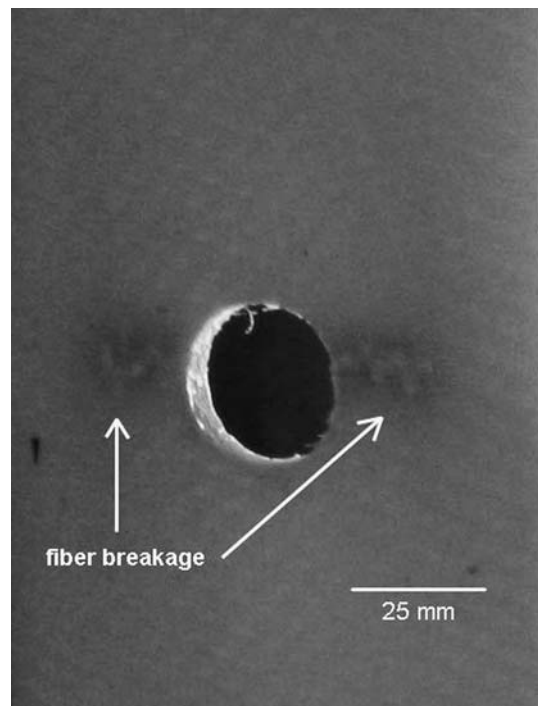


**Fig. 3** Damage parameter,  $D'$ , versus life ratio master curve for all notched specimens tested and all stress levels

specimen at a life ratio of 0.99, or just prior to final fracture.

**Conclusion**

This work demonstrates that the proposed damage parameter  $D'$ , obtained readily from the in-situ



**Fig. 4** Damage at the hole edge in a specimen at a life ratio of 0.99,  $N = 64,800$  (peak cyclic stress level of 115 MPa)

monitoring of loads and displacements can be used to predict life from a master curve obtained as outlined in this paper. Further work hopes to extend this approach to life prediction under cumulative damage conditions and variable block loading.

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